

Single particle spectra from information theory point of view

F.S. Navarra¹, O.V. Utyuzh², G. Wilk³ and Z. Włodarczyk⁴

¹*Instituto de Física, Universidade de São Paulo,*

Caixa Postal 66318, 05389-970 São Paulo, Brazil,

e-mail: navarra@if.usp.br

²*The Andrzej Sołtan Institute for Nuclear Studies,*

Hoża 69, 00681 Warsaw, Poland,

e-mail: utyuzh@fuw.edu.pl

³*Institute of Physics, Świętokrzyska Academy,*

Świętokrzyska 15, 25-405 Kielce, Poland,

e-mail: wlod@pu.kielce.pl

February 1, 2008

Abstract

It is demonstrated how to obtain the least biased description of the single particle spectra measured in all multiparticle production processes by using information theory approach (known also as MaxEnt approach). The case of e^+e^- annihilation in hadrons process is discussed in more detail as an example. Comparison between MaxEnt approach and simple dynamical model based on the cascade process is presented as well.

Information theory is nowadays widely used in all branches of sciences to provide the *least biased, most probable* description of data for which we have only limited amount of *information* [3]. The information is defined by means of information entropy, which, in turn, can assume extensive (Shannon) or nonextensive (Tsallis) form, depending on circumstances (see [3, 6, 8, 4] for details and further references). In our case where we are interested in single particle rapidity distributions, dN/dy , maximization of such entropy (known as MaxEnt procedure [3, 8, 4]) with constraint given by the energy conservation leads to the following form of the probability distribution to find one of the N produced particles of transverse mass $\mu_T = \sqrt{\mu^2 + \langle p_T \rangle^2}$ in the longitudinal phase space described by rapidity y such that energy of this particle is $E = \mu_T \cosh y$ and its longitudinal momentum is $p_L = \mu_T \sinh y$:

$$p(y) = \frac{1}{N} \frac{dN}{dy} = \frac{1}{Z_q} \exp_q(-\beta_q \cdot \mu_T \cosh y). \quad (1)$$

This form is identical with that used in statistical models but now Z_q and β_q are no longer *free parameters* to be fitted when comparing with experimental data but instead are given by the normalization condition and energy conservation constraint¹,

$$\int_{-Y_m}^{Y_m} dy p(y) = 1 \quad \text{and} \quad \int_{-Y_m}^{Y_m} dy \mu_T \cdot \cosh y \cdot [p(y)]^q = \frac{\kappa_q \cdot W}{N} \quad (2)$$

(where $\pm Y_m$ are maximal rapidities available in rest frame of hadronizing source, see [8, 4] for details). As is demonstrated in [4, 5] with $q > 1$ (responsible for dynamical fluctuations, see [9, 10] and q -inelasticity κ_q (connected to true inelasticity K_q defining a fraction of the original available energy W used for the production of secondaries, $K_q = \kappa_q/(3 - 2q)$, cf. [4]) as the only parameters we can describe all data for pp and $p\bar{p}$ collisions as well as recent data for nuclear collisions. Here we show in Fig. 1 that we can also fit fairly well e^+e^- annihilation data for which, by definition, $K_q = 1$ (the whole energy must be used for the production, there are no leading particles) and q remains the only free parameter. It turns out that in this

¹ $1 = \frac{1}{N} \int_{-Y_m}^{Y_m} dy p(y) = \frac{1}{N} \int_{-Y_m}^{Y_m} dy \frac{1}{Z_q} \exp_q(-\beta_q \mu_T \cosh y) = \frac{1}{N Z_q} \int_{-Y_m}^{Y_m} dy \exp_q(-\beta_q \mu_T \cosh y)$

case $q < 1$ ². This fact indicates (see [10]) that in e^+e^- processes (where $K_q = 1$) one cannot obtain equilibrium state because the corresponding temperature parameter depends now on energy: $T = 1/\beta_q = T_0 - (1 - q)E$ and system is highly influenced by the conservation of energy constraint. In Fig. 1 we have also shown attempts to fit the same data using the well defined sequential decay (cascading) hadronization developed in [7] instead of instantaneous hadronization³. Cascade here is defined by the sequence of decay processes, $M_l \rightarrow [M_{l+1}^{(1)} = k_1 M_l] + [M_{l+1}^{(2)} = k_2 M_l]$ (proceeding until $M_l > 2\mu_T$) in which all dynamics is summarily described by distribution $P(k)$ of decay parameters $k_i \in (0, 1)$ (such that in each vertex $k_1 + k_2 < 1$, see [7] for details). It is easy to realise that in this case dN/dy is given by a kind of random walk in rapidity space, which, in turn, results in its gaussian-like shape (cf. Fig. 1). However, one cannot find any reasonable values of decay parameters k_i to fit data with such distribution, whereas MaxEnt in its nonextensive version can do it with $q = 0.6$. The remaining small discrepancies for small and large rapidities indicate therefore a need for some additional information to be incorporated here (see, for example, discussion in [5]). When introduced it could then be again checked against experimental data. In this way we could always obtain good description of data with only *minimal* number of assumptions, i.e., with minimal information content [3, 8]. This is the main advantage of the method presented here and in our opinion it deserves further detailed investigations by using wide spectrum of the available multiparticle production data of all kinds.

²To be contrasted with $q > 1$ values needed to describe the p_T distributions in the same processes instead [2].

³Cascade could be regarded as a viable alternative because of the *a priori* cascade character of the quark \rightarrow gluon and gluon \rightarrow quark-antiquark processes preceding hadronization expected to be present in e^+e^- annihilations [1].

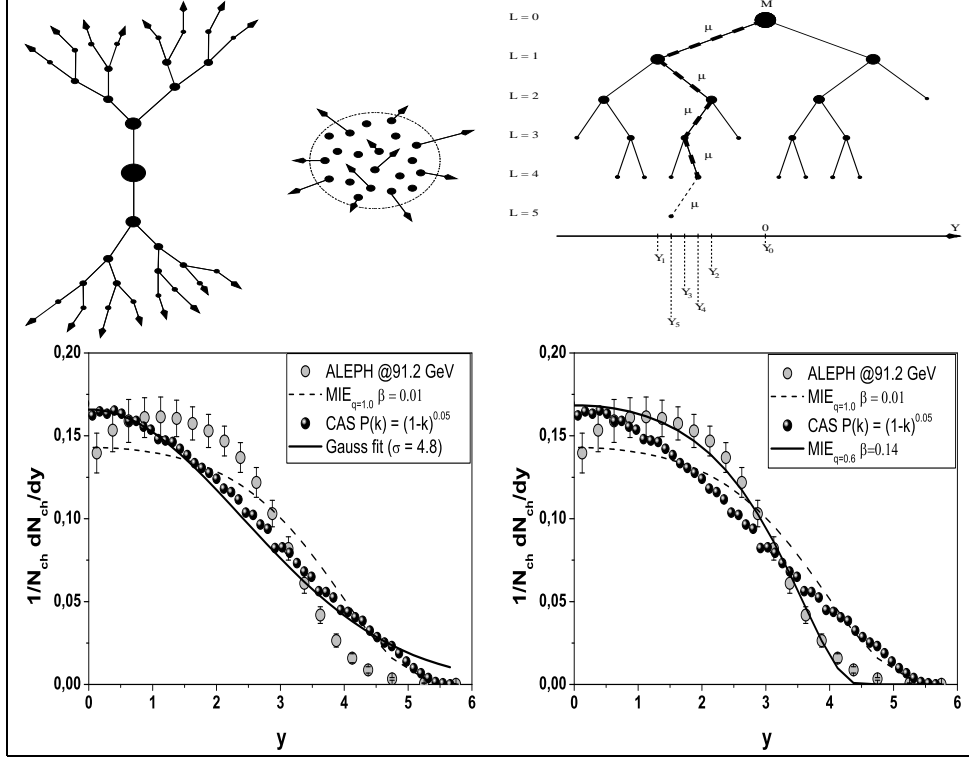


Fig.1 Upper-left panel: visualization of differences between sequential cascade hadronization (CAS) [7] and instantaneous one described by MaxEnt [8]. Upper-right panel: closer look at the way dN/dy arises in CAS (resembling random walk in y -space). Lower panels: results of calculations of dN/dy using CAS and MaxEnt and compared with e^+e^- annihilation data by ALEPH [1]. Left panel - comparison with CAS model giving the same multiplicity [7] (it can be extremely well approximated by gaussian fit) and with MaxEnt for $K_q = 1$ and $q = 1$ (it does not fit data). Right panel - gaussian fit has been replaced by MaxEnt fit with total inelasticity $K_q = 1$ and with varying q ; for $q = 0.6$ we obtain quite good agreement with data.

References

- [1] Barate R et al. (ALEPH Collab.) (1998) Studies of Quantum Chromodynamics with the ALEPH detector. Phys. Rep. 294:1-165.
- [2] Bediaga I, Curado E and de Miranda J (2000) A nonextensive thermodynamical equilibrium approach in $e^+e^- \rightarrow$ hadrons. Physica A286:156-163.

- [3] Harremoës P, Topsøe F (2001) Maximum Entropy Fundamentals. *Entropy* 3:191-226.
- [4] Navarra F, Utyuzh O, Wilk G, Włodarczyk Z (2003) Estimating the inelasticity with the information theory approach. *Phys Rev D* 67:114002-1/10.
- [5] Navarra F, Utyuzh O, Wilk G, Włodarczyk Z (2003) Information theory approach (extensive and nonextensive) to high energy multiple production processes. In: *Proc. of NEXT2003*, Villasimius (Cagliari), 21-28 September 2003; hep-ph/0312052.
- [6] Tsallis C (2000) Nonextensive Statistical Mechanics and Thermodynamics: Historical Background and Present Status. *Lecture Notes in Physics LPN560*:3-80.
- [7] Utyuzh O, Wilk G, Włodarczyk Z (2000) On the possible space-time fractality of the emitting source. *Phys Rev D* 61:034007-1/8.
- [8] Wilk G, Włodarczyk Z (1991) Information Theory and Phenomenology of Multiple Hadronic Production. *Phys Rev D* 43:794-801.
- [9] Wilk G, Włodarczyk Z (2000) Interpretation of the nonextensivity parameter q in some applications of Tsallis statistics and Lévy distributions. *Phys. Rev. Lett.* 84:2770-2773.
- [10] Wilk G, Włodarczyk Z (2001) The imprints of nonextensive statistical mechanics in high energy collisions. *Chaos, Solitons and Fractals* 13/3:581-594.